Preliminary Calculations of Aerodynamic, Thrust and Power Characteristics, Design, Analysis and Research Corp., Lawrence, KS, 1990, pp. 383–400.

[6]Hoak, D. E., "USAF Stability and Control Datcom," U.S. Air Force Wright Aeronautical Labs., AFWAL-TR-83-3048, Wright–Patterson AFB, OH, Oct. 1960 (rev. 1978).

[7]Perkins, C. D., and Hage, R. E., "Estimation of Airplane Dihedral Effect," *Airplane Performance Stability and Control*, Wiley, New York, 1949, pp. 343–346.

[8]Prandtl, L., "Tragflügel Theorie," *Nachricten von der Gesellschaft der Wissenschaften zu Göttingen*, Geschäeftliche Mitteilungen, Klasse, 1918, pp. 451–477.

[9]Prandtl, L., "Applications of Modern Hydrodynamics to Aeronautics," NACA TR-116, June 1921.

[10]Phillips, W. F., "Modern Adaptation of Prandtl's Classic Lifting-Line Theory," *Journal of Aircraft*, Vol. 37, No. 4, 2000, pp. 662–670.

[11]Anderson, J. D., "Incompressible Flow over Finite Wings: Prandtl's Classical Lifting-Line Theory," *Fundamentals of Aerodynamics*, 3rd ed., McGraw–Hill, New York, 2001, pp. 360–387.

[12]Bertin, J. J., and Smith, M. L., "Incompressible Flow About Wings of Finite Span," *Aerodynamics for Engineers*, 3rd ed., Prentice–Hall, Upper Saddle River, NJ, 1998, pp. 261–336.

[13]Katz, J., and Plotkin, A., "Finite Wing: The Lifting-line Model," *Low-speed Aerodynamics, from Wing Theory to Panel Methods*, McGraw–Hill, New York, 1991, pp. 193–212.

[14]Glauert, H., *The Elements of Aerofoil and Airscrew Theory*, 2nd ed., Cambridge Univ. Press, Cambridge, England, U.K. 1959, pp. 142–145.

[15]Multhopp, H., "Die Berechnung der Auftriebs Verteilung von Tragflugeln," *Luftfahrtforschung*, Vol. 15, No. 14, 1938, pp. 153–169.

[16]Rasmussen, M. L., and Smith, D. E., "Lifting-Line Theory for Arbitrary Shaped Wings," *Journal of Aircraft*, Vol. 36, No. 2, 1999, pp. 340–348.

[17]Lotz, I., "Berechnung der Auftriebsverteilung beliebig geformter Flugel," *Zeitschrift für Flugtechnik und Motorluftschiffahrt*, Vol. 22, No. 7, 1931, pp. 189–195.

[18]Karamcheti, K., "Elements of Finite Wing Theory," *Ideal-Fluid Aerodynamics*, Wiley, New York, 1966, pp. 535–567.

Viscous Compressible Flow Through a Hole in a Plate, Including Entry Effects

P. M. Galluzzo* and H. Babinsky†
University of Cambridge,
Cambridge, England CB2 1PZ, United Kingdom
and
G. R. Inger‡
Iowa State University, Ames, Iowa 50011-3231

Nomenclature

A	=	hole cross-sectional area
A_{plate}	=	total surface area of porous plate
D	=	nominal circular diameter of hole
k_D	=	scaling factor for diameter of each hole, due to roughness (≤ 1)
$k_{porosity}$	=	scaling factor for the porosity of each plate, due to blocked holes (≤ 1)
ℓ_i	=	entry length, defined as distance along the axis of a hole where the center-line velocity reaches 99% of its fully developed value
M	=	local Mach number in hole
$\dot{m}$	=	total transpiration mass flow through porous plate
p	=	static pressure
p_0	=	stagnation pressure
R	=	ideal gas constant
Re	=	hole Reynolds number based on diameter
Re_{eff}	=	effective Reynolds number of flow inside the hole
T	=	absolute static temperature, assumed constant along hole
t	=	plate thickness
V	=	cross sectionally averaged flow velocity in hole
x	=	axial coordinate inside hole
γ	=	specific heat ratio
Δp_{bubble}	=	pressure drop across the separation bubble at hole entrance
κ	=	ratio of apparent skin-friction coefficient in entry length to the skin-friction coefficient of fully developed pipe flow
μ	=	absolute viscosity
ρ	=	cross sectionally averaged density
$\bar{\tau}_w$	=	apparent wall skin friction, averaged over axial length

Subscripts

1	=	hole entrance station
2	=	hole exit station

Received 10 December 2001; revision received 8 February 2002; accepted for publication 15 February 2002.

*Graduate Research Student, Department of Engineering.
†Senior Lecturer, Department of Engineering. Member AIAA.
‡Professor, Department of Aerospace Engineering and Engineering Mechanics. Associate Fellow AIAA.

Introduction

WALL transpiration through porous surfaces is used in many aerodynamic flow control devices as a means of influencing boundary-layer behavior. The use of plates with microscopic laser-drilled holes has come to be preferred for this purpose. To predict the performance of such plates one must have an accurate knowledge of how the transpiration mass flux depends on relevant parameters such as pressure difference, hole diameter, and fluid properties. The transpiration mass flux is related to the fluid velocity inside each hole as

$$\dot{m} = (\rho_2 V_2) \times A_{plate} \times \text{porosity} \tag{1}$$

Inger and Babinsky[1] have recently proposed a new theory to predict the mass flux vs pressure difference relationship, based on a solution of the continuity equation, the momentum equation, and the ideal gas equation, for each hole independently, that is, a large distance is assumed to exist between holes.

Near the entrance to each hole of a porous plate, the flow is not yet fully developed, and the velocity gradient near the walls is quite steep. This will increase the skin friction near the wall and, by altering the velocity profile of the flow, cause a change in the momentum of the flow. Inger and Babinsky[1] assumed that the average value of skin friction over the entry length ℓ_i is some constant κ greater than its fully developed value of $(16/Re) \times \frac{1}{2}\rho V^2$ and that the average value of skin friction over the entire length of the hole is given by

$$\begin{aligned} \bar{\tau}_w \Big/ \tfrac{1}{2}\rho V^2 &= (16/Re)[(t - \ell_i)/t] + \kappa(16/Re)(\ell_i/t) \\ &= (16/Re)[1 + (\kappa - 1)(\ell_i/t)] \end{aligned} \tag{2}$$

Inger and Babinsky[1] proposed that the value of κ be chosen empirically to match experimental data. They assumed also that by manipulating κ in this way the effects of irregularities in the holes of a porous plate, such as roughness and sharp-edged intakes, could be accommodated for. However, it will be shown here that an analytical value for κ may be calculated for nearly all cases and that nonideal aspects of the hole geometry can be dealt with separately.

Proposed Improvements to Skin-Friction Model

When the flow enters each hole it is not yet fully developed; initially, a boundary layer develops along the wall of the hole, causing the skin friction to be larger than the value for fully developed pipe flow by some factor κ [Eq. (2)]. Inger and Babinsky[1] recommended a value for κ of between 2 and 3, giving κ a dual role: On one hand, it was designed to capture entry effects; on the other hand, it was required to be the sole method of compensating for manufacturing inaccuracies.

Inger and Babinsky's[1] analysis used the momentum equation

$$\rho A V \,\mathrm{d}V = -A\,\mathrm{d}p - \pi D \tau_w \,\mathrm{d}x \tag{3}$$

However, when the velocity is averaged, the momentum term ($\rho A V \,\mathrm{d}V$) in Eq. (3) now fails to capture the change in momentum due to the changing velocity profile (the value of which is $\pi \rho V^2 D^2/12$). Hence, when $\mathrm{d}m_{vp}$ is denoted as a small change in momentum due to the changing velocity profile,

$$\rho A V \,\mathrm{d}V + \mathrm{d}m_{vp} = -A\,\mathrm{d}p - \pi D \tau_w \,\mathrm{d}x \tag{4}$$

Thus, with Eq. (3) in mind, $\bar{\tau}_w$ is redefined as

$$\bar{\tau}_w = \underset{0<x<t}{\text{average}} \left\{ \tau_w + \frac{1}{\pi D}\frac{\mathrm{d}m_{vp}}{\mathrm{d}x} \right\}$$

$$= \underset{0<x<t}{\text{average}} \left\{ -\frac{D}{4}\left(\rho V \frac{\mathrm{d}V}{\mathrm{d}x} + \frac{\mathrm{d}p}{\mathrm{d}x} \right) \right\} \tag{5}$$

Accordingly, the definition of κ will also now need to be clarified. To do this, it is first assumed that the flow inside the hole near the entrance of the hole is incompressible. Despite that compressibility is important when considering the overall flow,[1] this assumption is justified for two reasons:

1) The effects of non-fully developed flow (such as changing velocity profile) are largely independent of compressibility.

2) The density does not change very much over the entry length when $\ell_i \ll t$ is assumed.

With this assumption in place, V must be constant in the entrance region. Hence, when the form of Eq. (5) is noted, κ is (re)defined as the ratio of the pressure drop across the entrance region to the pressure drop across an equal length of fully developed flow:

$$\kappa = \frac{(\Delta p)_{\text{entry length}}}{(\Delta p)_{\text{fully developed}}} = \frac{\int_0^{\ell_i} (-\mathrm{d}p/\mathrm{d}x)\,\mathrm{d}x}{\ell_i \left(\frac{1}{2}\rho V^2 \big/ D/4\right)(16/Re_1)}$$

$$= \frac{(\Delta p)_{\ell_i}}{32(\ell_i/DRe_1)\rho V^2} \tag{6}$$

This may be thought of as the ratio of the apparent friction coefficient in the entrance region ($0 < x < \ell_i$) to the friction coefficient of fully developed Poiseuille flow (equal to $16/Re$). Equation (3) allows the determination of κ based on the results of other investigators who have calculated Δp in the entry length. Note that Eq. (2) is still applicable.

Inger and Babinsky[1] have proposed that the flow everywhere in the hole may be treated as isothermal and that the conditions for Eq. (6) are that in the entrance region the flow is incompressible. For these flow conditions, a vast amount of research (e.g., Refs. 2–5) has been performed over the last 100 years to calculate analytical solutions. We may conveniently use the results of these analytical investigations to determine the value of κ.

Unfortunately, every analytical solution for the (isothermal and incompressible) flow inside the entrance to a circular hole gives its own unique values both for entry length ℓ_i and pressure drop Δp at this position. To be able to compare these results, a common value for ℓ_i must be selected so that κ may be evaluated at this new position for each analytical model. Friedmann et al.[6] numerically obtained a precise value for the entry length of $\ell_i = 0.056DRe_1$, where D is the hole diameter and Re_1 is the inlet Reynolds number based on hole diameter. They report that this value is accurate to within $\pm 10\%$ for Reynolds numbers above 35, embracing nearly all practical situations.

Denoting $(\ell_i)_j$ and $(\Delta p)_j$ as the values proposed by various investigators for entry length and the pressure drop at this distance

Table 1 Values for κ from different inlet length models

Author	$(\ell_i)_j/(DRe_1)$	$(\Delta p)_j \big/ \frac{1}{2}\rho V^2$ at $(\ell_i)_j$	κ at $0.056DRe_1$
Schiller[2]	0.0285	2.984	1.324
Campbell and Slattery[3]	0.0610	5.083	1.329
Langhaar[4]	0.05675	4.911	1.357
Fargie and Martin[5]	0.0488	4.460	1.373

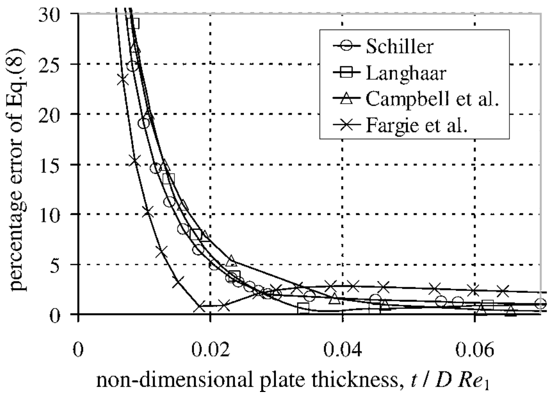

Fig. 1 Percentage error of Eq. (8) at a range of plate thicknesses.

and $0.056DRe_1$ as the correct value for ℓ_i, we obtain the following relation for κ in terms of $(\Delta p)_j$ and $(\ell_i)_j$ by using Eqs. (2) and (5):

$$\frac{D}{4(\ell_i)_j}\frac{(\Delta p)_j}{\frac{1}{2}\rho V^2} = \frac{16}{Re_1}\left[1 + (\kappa - 1)\frac{0.056DRe_1}{(\ell_i)_j}\right]$$

$$\Rightarrow \kappa = 1 + 0.279\left(\frac{(\Delta p)_j}{\frac{1}{2}\rho V^2}\right) - \frac{(\ell_i)_j}{0.056DRe_1} \tag{7}$$

Table 1 illustrates that the various analytical entry models agree quite closely when treated in this way. Taking the average (1.346) and substituting into Eq. (2), we find

$$\bar{\tau}_w \big/ \tfrac{1}{2}\rho V^2 = 16/Re[1 + 0.019(ReD/t)] \tag{8}$$

The subscript 1 has been dropped from the second of the two Reynolds numbers in Eq. (8) because Inger and Babinsky[1] assume that the flow inside each hole is isothermal; this means that the Reynolds number cannot vary along the length of each hole.

Each of the aforementioned analytical entry-length models was used to test the accuracy of this κ model, that is, Eq. (8), and the percentage error of Eq. (8) was calculated for a range of plate thicknesses. This percentage error is plotted in Fig. 1 for each entry-length model.

All of the models used in Fig. 1 are seen to agree very closely and, hence, can be relied on to give a good indication of the accuracy of Eq. (8). Clearly, Eq. (8) has a negligible error for $t > 0.05DRe_1$ and is accurate to within 10% for plate thicknesses as small as $0.015DRe_1$. The tendency of Eq. (8) to overpredict increasingly the value for κ as the thickness is decreased toward zero is to be expected because by nature it assumes that all entry effects are completed by the time the flow leaves the hole.

Proposed Methods of Compensating for the Nonideal Hole

Based on analysis of previous research into the entry length of circular tubes, it is suggested to take κ as 1.34 and ℓ_i as $0.056DRe$, yielding Eq. (8). However, because κ is now based on its analytical value for entry flow into idealized circular holes, an additional strategy is required to model the small manufacturing inaccuracies invariably present in practise. (Inger and Babinsky[1] empirically chose κ specially for this purpose.)

1) A common problem with porous plates is that dirt can block the holes and in practice plates are often found to be less porous than

specified. Although this does not affect the flow through a single (unblocked) hole, the plate porosity must be scaled [see Eq. (10)] by a factor k_{porosity}, where $k_{\text{porosity}} \leq 1$.

2) For a given cross-sectional area, a perfect circular cross section gives the smallest viscous pressure drop per unit length. Roughness represents a departure from this ideal, whereby the fluid in and amongst the protuberances on the surface will be slowed from the nominal velocity (given by the standard parabolic velocity distribution). Consequently, away from the wall, the velocity will be above its nominal value on average, and so we may say that roughness reduces the effective cross-sectional area of the holes. This could be reflected [see Eq. (9)] in an empirical scaling factor k_D, for the diameter of the hole (where $k_D \leq 1$). The entry length ℓ_i is proportional to D^2; for this reason, the entry length in Eq. (9) is scaled by k_D^2. A typical value for k_D (based on typical modern laser-drilled holes) is around 0.95.

3) The sharp-edged intake is likely to cause a separation bubble. One consequence of this is that the value of ℓ_i in Eq. (8) may need to be reduced due to improved mixing. However, another more serious consequence of a separation bubble is that an additional pressure drop Δp_{bubble} will occur across the separation bubble, quite separately from the viscous pressure drop in the remainder of the hole. Ward-Smith[7] suggests a value $\Delta p_{\text{bubble}} = 0.25\rho_1 V_1^2$, which would need to be added to the pressure drop due to entry effects and viscous drag.

Modifications for Inger and Babinsky[1] Porous Plate Theory

If Eq. (8) and the recommendations from the last section are incorporated into the Inger–Babinsky theory,[1] the Mach number at exit becomes

$$M_2 = \frac{V_2}{\sqrt{\gamma RT}} = \frac{\sqrt{1 + 2Re_{\text{eff}}^2\left[(p_1/p_2)^2 - 1\right]\ell n(p_1/p_2)} - 1}{2\sqrt{\gamma}Re_{\text{eff}}\,\ell n(p_1/p_2)} \tag{9}$$

where

$$Re_{\text{eff}} = \frac{\rho_2 (k_D D)^2 \sqrt{RT}}{32\mu t} \frac{1}{1 + 0.019\left[\rho_1 V_1 (k_D D)^2 / \mu t\right]}$$

and

$$p_1 = \frac{p_0}{\left(1 + (\gamma - 1)/2M_1^2\right)^{\gamma/(\gamma - 1)}} - \Delta p_{\text{bubble}}$$

The expression for transpiration mass flux through the porous plate given in Eq. (1) is altered to yield

$$\dot{m} = (\rho_2 V_2) \times A_{\text{plate}} \times [k_{\text{porosity}} \times (\text{nominal porosity})] \tag{10}$$

Comparison of Theory and Experiment

Experimental apparatus, methods and error sources are described by Inger and Babinsky.[1]

Figure 2 shows the predictions from Eq. (9), with $k_{\text{porosity}} = 0.88$, $k_D = 0.95$, and $\Delta p_{\text{bubble}} = 0.25\rho_1 V_1^2$, alongside the experimental results with representative error bars. The values of k_{porosity} and k_D were chosen to be physically as realistic as possible. The height of the roughness in the holes of the porous plates used in the experiment was seen to be of the order of $0.05D$, and the plates were seen to be 12% blocked. Clearly, there is a very close agreement between Eq. (9) and experiment, and because the entry length effects

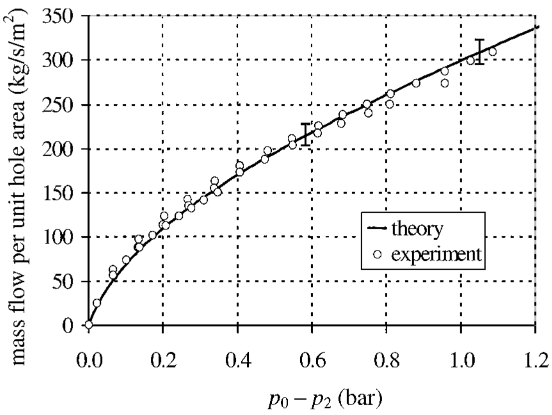

Fig. 2 Comparison of theory and experiment.

have now been treated in a physically sound manner, Eq. (9) may be expected to apply accurately to a vast range of practical situations.

In contrast with Eq. (9), the Inger–Babinsky model[1] fails to make use of the k_{porosity} factor to account for blocked holes. In place of that factor, the value of κ was artificially increased until theoretical and experimental results matched. As a result of this, two major advantages of Eq. (9) over the Inger–Babinsky model can be identified. The first advantage is that κ no longer has to be chosen from a wide range of values for every practical situation (Inger and Babinsky recommend the range $\kappa = 2 \rightarrow 3$); the correct value is 1.34. The second advantage of the new approach [Eq. (9)] is that it identifies all significant physical parameters and so can be expected to successfully predict a far wider range of practical conditions than the Inger–Babinsky model.

Conclusions

The pressure drop in the entrance region of a hole from a porous plate has been analytically described using a physically realistic model, and the effects of a nonideal hole have been modeled separately. Experimental results were obtained that show very close agreement with the resulting expression for transpiration mass flux. The new theory is expected to apply accurately to a large range of practical situations because it is based entirely on basic physical principles.

References

[1] Inger, G. R., and Babinsky, H., "Viscous Compressible Flow Across a Hole in a Plate," *Journal of Aircraft*, Vol. 37, No. 6, 2000, pp. 1028–1032.

[2] Schiller, L., "Die Entwichlung der laminaren Geschwindigkeitsverteilung und ihre Bedeutung für Zähigkeitsmessungen," *Zeitschrift für Angewandte Mathematik und Mechanik*, Vol. 2, No. 2, April 1922, pp. 96–106.

[3] Campbell, W. D., and Slattery, J. C., "Flow in the Entrance to a Tube," *Transactions of the American Society of Mechanical Engineers*, Vol. 85, Ser. D, No. 1, 1963, pp. 41–46.

[4] Langhaar, H. L., "Steady Flow in the Transition Length of a Straight Tube," *Journal of Applied Mechanics*, Vol. 9, No. 2, June 1942, pp. A-55–A-58.

[5] Fargie, D., and Martin, B. W., "Developing Laminar Flow in a Pipe of Circular Cross-Section," *Proceedings of the Royal Society of London, Series A: Mathematical and Physical Sciences*, Vol. A321, No. 1547, March 1971, pp. 461–476.

[6] Friedmann, M., Gillis, J., and Liron, N., "Laminar Flow in a Pipe at Low and Moderate Reynolds Numbers," *Applied Scientific Research*, Vol. 19, No. 1, 1968, pp. 426–438.

[7] Ward-Smith, A. J., *Internal Fluid Flow: The Fluid Dynamics of Flow in Pipes and Ducts*, Oxford Univ. Press, Oxford, 1980, p. 376.